

# A network-based perspective on coherent structure detection from very-sparse Lagrangian data

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## Abstract

Coherent structure detection (CSD) is a long-lasting issue in fluid mechanics research as the presence of spatio-temporal coherent motion enables simpler ways to characterize the flow dynamics. Such reduced-order representation, in fact, has significant implications for the understanding of the dynamics of flows, as well as their modeling and control (Hussain, 1986). While the Eulerian framework has been extensively adopted for CSD, Lagrangian coherent structures have recently received increasing attention, mainly driven by advancements in Lagrangian flow measurement techniques (Haller, 2015; Hadjighasem et al., 2017). Lagrangian particle tracking (LPT), in particular, is widely used nowadays due to its ability to quantify fluid-parcel trajectories in three-dimensional volumes (Schanz et al., 2016).

Differently from Eulerian coherent structures – that are mainly based on velocity gradient tensor (Chen et al., 2015), thus requiring higher data resolutions –, the Lagrangian framework provides a more suitable representation to characterize sparse data. Accordingly, coherent motion detection in the Lagrangian viewpoint naturally fits real-world dataset availability such as, among others, biological flows (Tallapragada et al., 2011), atmospheric flows (Shnapp et al., 2019), as well as oceanic currents (Gould, 2001).

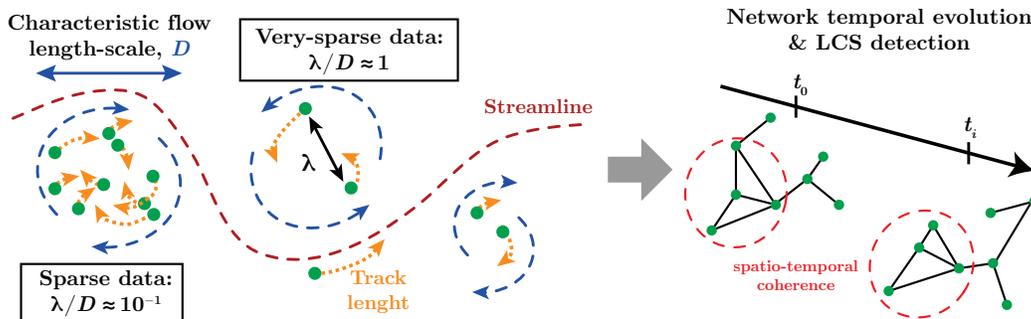


Figure 1: Schematic of the conceptual framework for CSD from sparse and very-sparse Lagrangian data. (left) Sketch of a vortical flow (blue dashed arrows) seeded with sparse and very-sparse tracers (green filled dots) and their tracks (orange dotted arrows);  $\lambda$  is the mean inter-particle distance. (right) Network evolution for CSD, where tracers correspond to nodes (green dots) while links are shown as black lines.

Several techniques have been proposed so far to identify coherent motion from Lagrangian data (Hadjighasem et al., 2017), including finite-time and finite-space Lyapunov exponents (Haller, 2015), trajectory complexity (Rypina et al., 2011), transfer operator-based methods (Ser-Giacomi et al., 2015), Lagrangian-averaged vorticity deviation, as well as fuzzy-clustering (Froyland and Padberg-Gehle, 2015), and spectral clustering (Schlueter-Kuck and Dabiri, 2017; Schneide et al., 2018; Martins et al., 2021). Although all these techniques have been tested to work reasonably well for 2D flows through densely-seeded particle distributions, they usually tend to fail in identifying the flow behavior for sparse or very-sparse data. In this regard, sparse and very-sparse Lagrangian data are characterized by extremely low particle densities per characteristic flow scale, namely  $O(10^{-1})$  and  $O(10^0)$  tracers per integral length scale in the flow, respectively (see Figure 1, left). Moreover, short track lengths contribute to the strong sparsity of the data.

Our work proposes a new network-based framework (Figure 1, right) that can be specifically suitable for very-sparse Lagrangian data from 3D experimental (realistic) flows. In particular, our methodology aims to exploit the instantaneous features of the trajectories (e.g., local geometry or local flow fields) to provide a spatio-temporal characterization of the flow. This procedure is in contrast with the classical *integral* approaches of previously-proposed techniques (as mentioned above), where the features of the whole trajectory are evaluated over an extended temporal interval. This operation, in fact, reduces the features of the whole trajectory to a scalar value at the starting or final particle position, thus leading to a very-sparse representation of the flow dynamics when very-sparse data are used. Moreover, our new perspective can be suitable for the analysis of unsteady flows, since the behavior of the particle trajectories are assessed over time.

Although very-sparse Lagrangian trajectories represent a challenge for CSD, they frequently appear in experiments. Accordingly, we have been carrying out experimental measurements on three-dimensional vortical flows to test our ideas based on the aforementioned network-based approach. In particular, particle tracking of seeded bubbles is currently in development to extract very-sparse trajectories that allows us to assess the effectiveness of the proposed method. Results related to the test case can hence pave the way for the analysis of a variety of other experimental data sets, so as to gain insight into realistic flows characterized by very-sparse Lagrangian trajectories. In conclusion, the combination of advances in LPT and the wide spectrum of possible applications, in conjunction with the growing development of network-based techniques in Fluid Mechanics (Iacobello et al., 2020), potentially makes our approach an effective alternative for CSD.

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